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Axion gauge symmetries and generalized Chern-Simons terms in $N = 1$ supersymmetric theories

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Abstract: We compute the extension of the lagrangian of $N = 1$ supersymmetric theories to the case in which some axion symmetries are gauged. It turns out that generalized Chern-Simons terms appear that were not considered in previous superspace formulations of general $N = 1$ theories.
Such gaugings appear in supergravities arising from flux compactifications of superstrings, as well as from Scherk-Schwarz generalized dimensional reduction in M-theory.
We also present the dual superspace formulation where axion chiral multiplets are dualized into linear multiplets.

Keywords: Supersymmetry and Duality, Supersymmetric Effective Theories.

Contents

1. Introduction

In many models of superstring and M-theory compactified to $D = 4$, it is possible to obtain a scalar potential which stabilizes some of the moduli (as well as matter) fields, lifting therefore the degeneracy of the moduli space of vacua. Examples of this phenomenon are type II superstrings compactified on orientifolds with NS and RR fluxes turned on [1]–[7], or Scherk-Schwarz generalized dimensional reductions [8] in M-theory.

In these theories the mass terms arise through a Higgs mechanism. The supergravity description corresponds to the gauging of some axion symmetries related to shifts of the scalar fields coming from wrapped RR forms or from the NS two-form B field in type II strings, and from the wrapped three-form in M-Theory.

In these gauge theories, generalized Chern-Simons terms emerge [9, 10, 11, 13, 14]. If the gauge groups are abelian they are of the form

$$\frac{2}{3} c_{AB,\,C} \int A^A \wedge A^C \wedge dA^B \,, \tag{1.1}$$

where $c_{AB,\,C}$ are real constants with symmetries

$$c_{AB,\,C} = c_{BA,\,C} \,, \qquad c_{AB,\,C} + c_{CA,\,B} + c_{BC,\,A} = 0 \,. \tag{1.2}$$

In the non abelian case an additional term is present

$$\frac{1}{4} c_{AB,\,C} f^{A}_{\ DE} \int A^D \wedge A^E \wedge A^C \wedge A^B \,, \tag{1.3}$$

where $f^{A}_{\ DE}$ are the structure constants of the gauge group.

In theories arising from type IIB compactification on $T_6/\mathbb{Z}_2$ orientifold [2, 3], the constants $c_{AB,\,C}$ are proportional to the RR and NS three-form fluxes F^{α}_{ABC} (with $A, B, C = 1, \ldots, 6$; $\alpha = 1, 2$), and equation (1.1) takes the form [15, 16]

$$\frac{2}{3} F^{\alpha}_{ABC} \int A^A_{\alpha} \wedge A^B_{\beta} \wedge dA^C_{\gamma} \epsilon^{\beta\gamma} \,, \tag{1.4}$$

Property (1.2) is understood in these theories from the fact that, in (1.4),

$$F^{[\alpha}_{ABC}\epsilon^{\beta\gamma]} = 0\,,$$

where the bracket $[\cdots]$ stands for complete antisymmetrization of the indices.

If we perform a Scherk-Schwarz dimensional reduction in M-theory with Scherk-Schwarz phase matrix M^A_B, the constants $c_{AB,C}$ come from the 5-d Chern-Simons form [12, 13]

$$d_{ABC}A^A \wedge F^B \wedge F^C \qquad (A, B, \ldots = 1, \ldots, 27)$$

and are given by

$$c_{AB,C} = d_{ABD}M^D_C\,.$$

In this case the condition (1.2) is a consequence of the fact that, in $N = 8$ $d = 5$ supergravity, d_{ABC} is an E_6 invariant tensor.

Another instance where a particular form of such terms appears is in deconstructed supersymmetric $U(1)$ gauge theories [17], where it arises for cancellation of mixed $U(1)$ anomalies [17, 18].

The occurrence of such terms was studied for $N = 2$ in ref. [9], but for a matter coupled $N = 1$ supersymmetric gauge theory they have not been considered previously.

It is the aim of the present investigation to give such completion for the $N = 1$ case. We will consider only the abelian gauge groups, as for example the groups of axion shift symmetries.

Let V^A be the superfield vector potentials and $W^A_\alpha = \bar{\mathcal{D}}^2\mathcal{D}_\alpha V^A$ denote the chiral supersymmetric field strengths, with $A = 1, \ldots, n_v$, where n_v denotes the number of vectors undergoing the gauging and α is a spinor index. The gauge transformations depend on chiral superfield parameters Λ^A,

$$V^A \longrightarrow V^A + \Lambda^A + \bar{\Lambda}^A\,, \qquad W^A \longrightarrow W^A\,.$$

Let the kinetic term of the vectors in the lagrangian be written as [19]

$$\int d^2\theta f_{AB}W^A W^B + \text{h.c.}\,, \tag{1.5}$$

(we have suppressed the contracted spinor indices) where the matrix f_{AB} is a holomorphic function of the scalar fields. We assume that under a gauge transformation (we will justify later this assumption) the kinetic matrix of the vectors transforms as

$$\delta_\Lambda f_{AB} = c_{AB,C}\Lambda^C$$

with $c_{AB,C}$ the real constants appearing in (1.1). Gauge invariance is achieved because of the presence of the generalized Chern-Simons terms which involve only the vector fields.

The axionic chiral multiplets can be dualized into linear multiplets [20]. The dual lagrangian exhibits Green-Schwarz couplings of the linear multiplets. Let b_i denote the two-form fields dual to the axion fields S^i and F^A the field strengths of the gauge potentials. If we write the gauge transformations of the dual axion multiplets as

$$\delta S^i = M^i_A\Lambda^A\,,$$

then the G-S coupling terms are the supersymmetric extensions of the bosonic dual terms

$$M^i_A b_i \wedge F^A \,.$$

The paper is organized as follows. In section 2 we recall the symplectic action of the σ-model isometries on vector fields as duality rotations and the need to introduce generalized Chern-Simons terms. In section 3 we derive the dual lagrangian with linear multiplets, whose physical bosonic components are antisymmetric tensors. Conclusions and outlooks are given in section 4. An appendix with some useful formulae is included.

2. Dualities, axionic symmetries and Chern-Simons terms

The standard form of the lagrangian density in $N = 1$ supersymmetric gauge theories is [19]

$$\int d^4\theta K(S, \bar{S}e^V) + \left[\int d^2\theta \left(f_{AB}(S)W^A W^B + P(S)\right) + \text{ h. c.}\right], \tag{2.1}$$

where K and P (the Kähler potential, and the superpotential respectively) are gauge invariant. The matrix f_{AB} is a chiral superfield, symmetric in the indices A, B, transforming in the twofold symmetric tensor product of the coadjoint representation of the gauge group, to make the action gauge invariant.

From the structure of the vector couplings [21] it follows that f_{AB} may have a more general transformation rule. The invariance of the system of field equations plus Bianchi identities [22] allows a transformation of f_{AB} in terms of a matrix of $\mathrm{Sp}(2n_v, \mathbb{R})$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \qquad A^tC, B^tD \text{ symmetric}, \quad A^tD - C^tB = 1 \tag{2.2}$$

of the form

$$if' = (C + Dif)(A + Bif)^{-1} \,. \tag{2.3}$$

The transformations of $\mathrm{Sp}(2n_v, \mathbb{R})$ mix electric and magnetic field strengths. When a subgroup of this symmetries becomes a local (gauge) symmetry, it must act on the gauge potentials so it must be *electric*; this means that necessarily $B = 0$. This implies $A^t = D^{-1}$ and

$$if' = CA^{-1} + (A^t)^{-1} if A^{-1} \,, \tag{2.4}$$

while the gauge vectors transform simply as $V' = AV$. Nevertheless, the gauge group may have, as embedded in $\mathrm{Sp}(2n_v, \mathbb{R})$, $C \neq 0$. The gauge group corresponds, in the scalar manifold, to a subset of the isometry group that has been gauged. A gauge transformation will result in a transformation of f (as a function of the scalar fields) of the type (2.4). If the symplectic transformation is constant, this will result in a change of the lagrangian as a total derivative[1], but if the transformation depends on local parameters, then the lagrangian (2.1) must be modified to achieve gauge invariance.

[1]Note that the convention taken in (2.3) is essential for this to be true

One can gauge abelian groups, which have $B = 0$, $A = 1$ and $C \neq 0$. They have a non trivial action on the scalar fields. Non abelian gaugings ($A \neq 0$) with $C \neq 0$ are also possible but will not be considered here.

From now on we will assume that the gauge group is abelian. We assume that we can choose local coordinates in the scalar manifold in such way that the set of chiral multiplets (n_c) can be split in two sets,

$$\{S^i\}_{i=1}^n \qquad \text{and} \qquad \{T^a\}_{a=1}^m\,, \qquad n + m = n_c\,,$$

in such way that only the multiplets S^i (together with the vector potentials) transform under the gauge group. We have

$$\delta_\Lambda T^a = 0\,, \qquad \delta_\Lambda S^i = M_A^i \Lambda^A\,, \qquad \delta_\Lambda V^A = \Lambda^A + \bar{\Lambda}^A\,. \tag{2.5}$$

Also, we assume that the matrix f_{AB} is of the form

$$f_{AB}(S,T) = d_{ABi} S^i + \tilde{f}_{AB}(T)\,.$$

Then

$$\delta_\Lambda f_{AB}(S,T) = c_{AB,\,C} \Lambda^C\,, \tag{2.6}$$

with

$$c_{AB,\,C} = d_{ABi} M_C^i\,. \tag{2.7}$$

We will see later that the lagrangian can be made gauge invariant if $d_{ABi} M_C^i$ are such that the second property in (1.2) holds.

The combination $S^i + \bar{S}^i - M_A^i V^A$ is gauge invariant, and the kinetic term for the chiral fields is of the form

$$\int d^4\theta K(T, S^i + \bar{S}^i - M_A^i V^A)\,.$$

(The fields S^i in this expression are the logarithms of what in (2.1) was denoted by S).

The relevant vector kinetic term is

$$\int d^2\theta d_{ABi} S^i W^A W^B + \text{h.c.}\,, \tag{2.8}$$

and its gauge variation is

$$\int d^2\theta c_{AB,\,C} \Lambda^C W^A W^B \; + \; \text{h. c..} \tag{2.9}$$

This term is a total derivative if $\Lambda^C(x,\theta)$ is an imaginary constant. Otherwise, to cancel this variation we must add a generalized Chern-Simons term, which can be constructed with the Chern-Simons multiplet introduced in ref. [20]. This term is

$$-\frac{2}{3} c_{AB,\,C} \int d^4\theta V^C \Omega^{AB}(V)\,, \tag{2.10}$$

with

$$\Omega^{AB}(V) = \mathcal{D}^\alpha V^{(A} W^{B)}_\alpha + \bar{\mathcal{D}}_{\dot\alpha} V^{(A} W^{\dot\alpha B)} + V^{(A} \mathcal{D}^\alpha W^{B)}_\alpha \,, \tag{2.11}$$

(the parenthesis $(\cdots)$ stands for symmetrization in the indices). Note that Ω is real; in particular, the last term is real because of the Bianchi identity

$$\mathcal{D}^\alpha W_\alpha = \mathcal{D}_{\dot\alpha} W^{\dot\alpha} = \overline{\mathcal{D}^\alpha W_\alpha}\,.$$

Under a gauge transformation the Chern-Simons multiplet transforms as

$$\delta_\Lambda \Omega^{AB}(V) = \mathcal{D}^\alpha(\Lambda^{(A} W^{B)}_\alpha) + \bar{\mathcal{D}}_{\dot\alpha}(\bar\Lambda^{(A} \bar W^{\dot\alpha B)})\,.$$

Using the property (proven in the Appendix)

$$V^{(C}\Omega^{AB)} = \frac{1}{6}\mathcal{D}^\alpha \left(V^A V^B W^C_\alpha + V^A V^C W^B_\alpha + V^B V^C W^A_\alpha\right) + \text{h.c.}\,,$$

the part in (2.10) which is symmetric in (A,B,C) gives zero contribution to the action, being a total space-time derivative. This means that if $c_{(AB,C)} \neq 0$ the variation (2.9) cannot be completely cancelled by a term like (2.10). So we require that $c_{(AB,C)} = 0$ as a consistency condition for gauge invariance.

The gauge variation of (2.10) is (see Appendix)

$$-c_{AB,C}\left(\int d^2\theta \Lambda^C W^A W^B + \text{h.c.}\right),$$

and we see that it cancels exactly the gauge variation of the vector kinetic term (2.9), so that the Chern-Simons-completed vector lagrangian

$$-\frac{2}{3}c_{AB,C}\int d^4\theta\, \Omega^{AB} V^C + \left(\int d^2\theta f_{AB} W^A W^B \ + \ \text{h.c.}\right)$$

is gauge-invariant.

This is in agreement with what was found in ref. [9] for the $N=2$ case.

Let us further observe that, in the Wess-Zumino gauge, the component expression of the Chern-Simons action (2.10) contains, beyond the bosonic contribution (1.1), the extra term

$$c_{AB,C}\bar\lambda^A \gamma^\mu \gamma_5 \lambda^B A^C_\mu\,.$$

This is needed in order to make gauge-invariant the fermionic contribution in (2.8) containing $\text{Im}\phi^i$ ($\phi^i = S^i|_{\theta=0}$)

$$d_{AB,i}\,\text{Im}\phi^i \partial_\mu \left(\bar\lambda^A \gamma^\mu \gamma_5 \lambda^B\right)$$

which then becomes, using (2.7),

$$-d_{AB,i}\left(\partial_\mu \text{Im}\phi^i - M^i{}_C A^C_\mu\right)\bar\lambda^A \gamma^\mu \gamma_5 \lambda^B\,.$$

3. Dual form of the lagrangian

The lagrangian studied in the previous section can be dualized by replacing the chiral multiplets S^i by the dual linear multiplets L_i. These are real multiplets satisfying the constraint $\mathcal{D}^2 L_i = \bar{\mathcal{D}}^2 L_i = 0$. In order to perform the dualization we introduce the real superfield U^i. The lagrangian connecting the two theories is

$$\begin{aligned}\mathcal{L} = & \int d^4\theta \left[K(T, U^i - M^i_A V^A) - L_i U^i + \left(d_{ABi} U^i - \frac{2}{3} c_{AB,C} V^C\right) \Omega^{AB}\right] + \\ & + \left[\int d^2\theta \tilde{f}_{AB}(T) W^A W^B \ + \ \text{h.c.}\right], \end{aligned} \tag{3.1}$$

with $c_{AB,C} = d_{ABi} M^i_C$. The original lagrangian is obtained by varying (3.1) with respect to L_i, which gives $U^i = S^i + \bar{S}^i$ (notice that L_i is not unconstrained), and substituting back in $\mathcal{L}$.

The dual lagrangian instead is obtained by varying with respect to U^i and substituting the equation obtained in $\mathcal{L}$. Let us define $\tilde{U}^i = U^i - M^i_A V^A$. Then, the relevant terms in (3.1) become

$$\int d^4\theta \left[K(T, \tilde{U}^i) - \tilde{U}^i (L_i - d_{ABi}\Omega^{AB}) - L_i M^i_A V^A + \frac{1}{3} c_{AB,C} V^C \Omega^{AB}\right].$$

Solving

$$\Psi_i \equiv \frac{\partial K(T, \tilde{U}^i)}{\partial \tilde{U}^i} - L_i + d_{ABi}\Omega^{AB} = 0$$

one gets

$$\int d^4\theta \left[\Phi(T, L_i - d_{ABi}\Omega^{AB}) - L_i M^i_A V^A + \frac{1}{3} c_{AB,C} V^C \Omega^{AB}\right],$$

where

$$K(T, \tilde{U}^i) - \tilde{U}^i (L_i - d_{ABi}\Omega^{AB}) = \Phi(T, L_i - d_{ABi}\Omega^{AB}) \quad \text{at} \quad \Psi_i = 0\,.$$

The gauge transformation of L_i are

$$\delta L_i = d_{ABi}\left[\mathcal{D}^\alpha \left(\Lambda^A W^B_\alpha\right) + \bar{\mathcal{D}}_{\dot\alpha}\left(\bar{\Lambda}^A W^{B\dot\alpha}\right)\right].$$

Notice that the variation of the Green-Schwarz term is now cancelled by the variation of the generalized Chern-Simons term (which has a different coefficient with respect to the dual formulation).

4. Conclusions

In this investigation we have given the superfield expression of the $N = 1$ lagrangian with gauged axion symmetries.

The lagrangian requires new coupling terms which were not present in the standard formulation because there it was assumed that the gauge transformations changed the kinetic matrix of the vectors as

$$f' = (A^t)^{-1} f A^{-1}\,,$$

where A is the adjoint action of the fields. It is interesting to observe that not all the axion gauge symmetries can be gauged, but only those for which the expression

$$d_{ABi} M_C = c_{AB,\,C} \qquad \text{satisfies} \qquad c_{(AB,\,C)} = 0\,.$$

In fact, the variation of the action with the term (2.10) included is

$$c_{(AB,\,C)} \int d^2\theta \Lambda^C W^A W^B \ + \ \text{h.c.}$$

and it vanishes only when the above consistency condition is fulfilled. The simplest case where $c_{(AB,C)} \neq 0$ is when we have only one axion S with coupling

$$S W^A W^B \delta_{AB}\,.$$

Under the axion symmetry $S \to S + \Lambda$ the lagrangian is not gauge-invariant at the classical level, rather it can be used to cancel (one-loop) quantum anomalies [23]–[25].

It is possible to extend the present analysis to the supergravity case and to non-abelian axion symmetries. Such cases are incountered in Scherk-Schwarz M-theory compactifications and in type IIB supergravity compactifications in the presence of fluxes.

In the non-abelian case ($A \neq \mathbb{1}$ in equation (2.2)), the coefficients $c_{AB,C}$ must satisfy the extra condition [9, 10]

$$f^D_{\ E(B} c_{A)D,F} - f^D_{\ F(B} c_{A)D,E} + \frac{1}{2} f^D_{\ EF} c_{AB,D} = 0\,. \tag{4.1}$$

This follows from the fact that for a vector transforming as

$$\delta \mathrm{V}_a = t_a{}^b{}_A \Lambda^A \mathrm{V}_b + C_{aA} \Lambda^A$$

the closure of the gauge algebra requires a cocycle condition on the coefficient C_{aA}[2]

$$t_a{}^b{}_A C_{bB} - t_a{}^b{}_B C_{bA} - f_{AB}^{\ C} C_{aC} = 0\,.$$

The condition (4.1) on the coefficients $c_{AB,C}$ is just the above relation, when specified to the twofold symmetric product of the coadjoint representation.

In the Wess-Zumino gauge, the supersymmetric version of the non abelian completion (1.3) is

$$c_{AB,C} f^B_{\ PQ} V^C \mathcal{D}^\alpha V^A \bar{\mathcal{D}}^2 \left(\mathcal{D}_\alpha V^P V^Q\right) + \text{h.c.}$$

[2] We thank R. Stora for enlightening discussions on this issue.

A. Some useful relations

Consider the superfield:

$$\Omega^{AB} = \mathcal{D}^\alpha V^{(A} W_\alpha^{B)} + \bar{\mathcal{D}}_{\dot\alpha} V^{(A} \bar{W}^{\dot\alpha B)} + V^{(A} \mathcal{D}^\alpha W_\alpha^{B)}\,, \tag{A.1}$$

where $(\cdots)$ stands form complete symmetrization in the indices.

Ω^{AB} is real thanks to the property

$$\mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot\alpha} \bar{W}^{\dot\alpha} \tag{A.2}$$

and it satisfies:

$$\bar{\mathcal{D}}^2 \Omega^{AB} = W^{(A\alpha} W_\alpha^{B)}\,. \tag{A.3}$$

Consider now the superfield $\Omega^{AB}V^C$. Its totally symmetric part can be written as a total derivative

$$\begin{aligned}\Omega^{(AB}V^{C)} &= \frac{1}{2}\mathcal{D}^\alpha \left(V^{(A}V^B W_\alpha^{C)}\right) + \text{h.c.}\\ &= \frac{1}{6}\mathcal{D}^\alpha \left(V^A V^B W_\alpha^C + V^B V^C W_\alpha^A + V^A V^C W_\alpha^B\right) + \text{h.c.}\end{aligned} \tag{A.4}$$

so that such a lagrangian term does not contribute to the action:

$$\int d^4x \int d^4\theta\, \Omega^{(AB}V^{C)} = 0\,. \tag{A.5}$$

Indeed, using (A.2), we have:

$$\begin{aligned}\Omega^{(AB}V^{C)} &= \mathcal{D}^\alpha V^{(A} W_\alpha^B V^{C)} + \frac{1}{2} V^{(A}\mathcal{D}^\alpha W_\alpha^B V^{C)} + \text{h.c.}\\ &= \frac{1}{2}\mathcal{D}^\alpha \left(V^{(A}V^{B}\right) W_\alpha^{C)} + \frac{1}{2} V^{(A}\mathcal{D}^\alpha W_\alpha^B V^{C)} + \text{h.c.}\\ &= \frac{1}{2}\mathcal{D}^\alpha \left(V^{(A}V^B W_\alpha^{C)}\right) + \text{h.c.}\end{aligned} \tag{A.6}$$

The gauge invariance. We explicitely show here, for the abelian case, that the lagrangian (2.1) completed with the Chern-Simons term (1.1) is gauge-invariant.

Indeed, from (2.5) we have

$$\delta_\Lambda \Omega^{AB} = \mathcal{D}^\alpha \left(\Lambda^{(A} W_\alpha^{B)}\right) + \text{h.c.}$$

Then

$$\begin{aligned}\delta_\Lambda \left(c_{AB,C}\int d^4\theta\, \Omega^{AB}V^C\right) &= c_{AB,C}\int d^4\theta \left[\Omega^{AB}\Lambda^C + \mathcal{D}^\alpha \left(\Lambda^A W_\alpha^B\right) V^C\right] + \text{ h.c.}\\ &= c_{AB,C}\int d^2\theta \left(W^A W^B \Lambda^C - \Lambda^A W^B W^C\right) + \text{h.c.}\,,\end{aligned} \tag{A.7}$$

where we have used (A.3) and the notation

$$W^A W^B = W^B W^A \equiv W^{\alpha A} W_\alpha^B = -W_\alpha^A W^{\alpha B}\,.$$

However, due to equations (1.2), we have

$$\begin{aligned} c_{AB,C}\Lambda^A W^B W^C &= (-c_{AC,B} - c_{BC,A})\Lambda^A W^B W^C \\ &= -c_{AB,C}\Lambda^A W^B W^C - c_{AB,C}\Lambda^C W^A W^B \end{aligned}$$

so that

$$c_{AB,C}\Lambda^A W^B W^C = -\frac{1}{2} c_{AB,C}\Lambda^C W^A W^B \,. \tag{A.8}$$

Using (A.8) in (A.7) we get

$$\delta_\Lambda \left(c_{AB,C} \int d^4\theta \Omega^{AB} V^C \right) = \frac{3}{2} c_{AB,C} \int d^2\theta \Lambda^C W^A W^B \; + \; \text{h.c.} \tag{A.9}$$

On the other hand, from (2.6), the gauge transformation of the vector kinetic term (1.5) is

$$\delta_\Lambda \left(\int d^2\theta f_{AB} W^A W^B \; + \; \text{h.c.} \right) = c_{AB,C} \int d^2\theta \Lambda^C W^A W^B + \text{h.c.} \tag{A.10}$$

so that the Chern-Simons-completed vector lagrangian

$$-\frac{2}{3} c_{AB,C} \int d^4\theta \Omega^{AB} V^C + \left(\int d^2\theta f_{AB} W^A W^B + \text{h.c.} \right)$$

is gauge-invariant.

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